The Limiting Constant of Gravitation. By Pliny E. Chase.

(Read before the American Philosophical Society, October 18, 1878.)

Newton and Lesage both thought that gravitation might be due to some action of an æther or "æthereal spirit." If such supposed action is uniform it should be capable of representation by some uniform or constant value, toward which planetary or gravitating motion should constantly tend.

Faraday sought in vain to find such a value, and his want of success led him to the belief that the "correlation of forces" could not include the force of gravity. It is true that a kind of constancy is observable in bodies at rest, and another kind in circular orbits; but if the distance from the principal center is changed, the former varies inversely as the square of the distance, the latter inversely as the square root of the distance. Inasmuch as there is no known limit of possible density, there is no obvious limit to the possible velocity of gravitating motion.

My various investigations have shown that heat, actinism, kinetic laws, spectral lines, the arrangement and masses of planets, interstellar nodes, barometric fluctuations, centers of inertia, terrestrial magnetism, chemical combinations, and the aggregation or dissociation of stellar systems, all point to the velocity of light as a limiting constant. Weber, Kohlrausch and Maxwell having found a like pointing, in the relations which exist between electro-static and electro-dynamic phenomena, it seems probable that the goal of Faraday's search may also have been the velocity of light, and that such velocity is the fundamental basis of universal correlation.

I have already pointed out three methods of approximation to the limit: 1, by the tendency towards equality in planetary revolution and in the mean moment of solar inertia of rotation; 2, by the tendency to equality between mean radial oscillatory velocity and the velocity which marks the limit between complete solar dissociation and incipient nucleal aggregation; 3, by the tendency to uniformity in dissociative velocity at each of the three principal centres of nebular condensation in the solar system.

Against the first of these methods the objection has been urged that it supposes the sun to be homogeneous. The validity of this criticism cannot be determined until the problem has been subjected to a rigid mathematical analysis. If such analysis should hereafter show that the objection is well taken, it may be found that the sun is more homogeneous than the dense planets, and sufficiently so to satisfy all the requirements of the method. Draper's recent photograph of the corona indicates a diameter twice as great as that of the sun. This is in exact accordance with the supposed gaseous nature of the sun, and, consequently, with its homogeneity, as well as with the relations which I have pointed out between Jupiter's mass and position.

Some have thought the second method faulty, because it involves a consideration of hypothetical conditions of nebular condensation, such as are inconsistent with the common notions of the nature of matter. But those conditions were introduced merely to indicate joint tendencies, without

any regard to the variety of possible or impossible forms which the tendencies may be supposed to assume or to indicate. In all mathematical physics an ideal completeness is assumed, such as is never found in nature. The method in question is analogous to the one which has been satisfactorily adopted in investigating the laws of elastic undulation.

It seems to have been generally admitted that the third method may be accepted as lending probability to the indications of the other two, but it involves the same question of dissociative velocity, and is, therefore, open to the same criticism as the second method. For this reason it seems desirable to see whether the problem can be successfully treated in some other way.

If gravitating movements have any common limit, either of originating efficiency or of ultimate tendency, which is uniform in all stellar systems, that limit should evidently be sought in the direction of phenomenal maxima, and with special reference to the principal center of the system. If the æthereal hypotheses are correct, we may reasonably presume that the gravitating constant is dependent upon some æthereal constant.

La Place established the general principle that the state of a system of bodies becomes periodic when the effort of the primitive conditions of movement has disappeared by the action of resistances. This principle, which is a necessary consequence of the third law of motion, is well illustrated in elliptical planetary orbits, in which the cyclical movement may be resolved into alternate oscillations, of approach to perihelion and retreat to aphelion. The duration of all such oscillations, whether circular, slightly elliptical, or as nearly radial and rectilinear as the central nucleus will allow, is determined by the length of the major axis, varying as the $\frac{3}{2}$ power of the length. If the major axes are equal, the oscillations are synchronous.

If orbital collisions of particles, in the neighborhood of the focus, shorten the major axes, cosmical rotation may be substituted for free planetary revolution. But the limiting value, which is to be alternately overcome and renewed, will not be changed thereby; the period for destroying or acquiring that limiting value should still be one-half of the cyclical period, or the period of a half rotation.

The equation of constant velocity, in an elastic atmosphere or in an æthereal medium, is

$$v = \sqrt{gh} = gt,$$

v denoting the wave-velocity; g, the acceleration of gravity at the point of observation; h, the modulus of elasticity, or the height of a homogeneous atmosphere; t, the time of rise or fall, through $\frac{1}{2}$ h, under the constant retardation or acceleration g; t is also, as has just been shown, the time of a half-rotation which is supposed to be due to æthereal impulses. Challis has found* that if all the ordinary central forces are due to transformed æthereal vibrations, "the actions of such forces on atoms are in every instance attributable to *æthereal currents*, whether the atoms be *immediately* acted upon by steady motions of the æther or by æthereal vibration."

* Phil. Mag., Sept., 1872; Sept., 1876; June, 1878.

The constancy of wave-velocity requires that h and t should vary inversely as g. The law of conservation of areas demands the same ratios of variability in the rotation of any contracting or expanding nebular nucleus; for, the velocity of rotation varying inversely as radius, and the distance traversed varying as radius, the time of rotation (or t, the time of semi-rotation) varies as the square of radius; but g varies inversely as the square of radius, $\therefore\ gt \propto \frac{1}{r^2} \times r^2$, and is constant for all possible stages of nebular condensation. The record of rotation is, therefore, invariable, representing the undulatory velocity of the æthereal medium, as well as the constant limiting velocity of gravitating tendency for which Faraday sought.

The value of g being a maximum, in our system, at Sun's surface, there is where the limiting value of gt is to be found. If we estimate Sun's semi diameter* at 16′ 2″, Earth's mean radius vector is 214.41 solar radii. Laugier's mean estimate of t (the time of Sun's semi-rotation) is 12.67 days, or 1093872 seconds; $\sqrt{gr} = (214.41^{\frac{3}{2}} \times 2\pi r) \div (365.256 \times 86400)$. $\therefore\ g = r \div 2559500$, and $gt = r \div 2.340$. But the velocity of light, according to Struve's constant of aberration, is $214.41\ r \div 497.825 = r \div 2.322$.† This investigation, therefore, leads to the same result as those which I have before undertaken, and gives *the velocity of light* as the limiting constant of gravitation.

Stated Meeting, November 1, 1878.

Present, 20 members.

Vice-President, MR. FRALEY, in the Chair.

Mr. J. B. Knight, Prof. L. Haupt, and Dr. Morris Longstreth, newly elected members, were introduced to the presiding officer and took their seats.

Letters accepting membership were received from Dr. Albert H. Smith, dated 1419 Walnut St., Phila., Oct. 20, 1878; Rev. Edward A. Foggo, D. D., 717 Locust St., Phila., Oct. 28, 1878; Rev. Samuel Longfellow, Germantown, Oct. 24; and Dr. A. S. Packard, Jr., Brown University, Providence, R. I., Oct. 18, 1878.

Letters of acknowledgment were received from the Observatory at Prag, Nov. 6, 1877 (99,100, List); the Royal Danish Academy, Sept. 30, 1878 (100, List); the Royal

* Amer. Nautical Almanac.

† This is equivalent to Faye's value of gt for lat. 16° 50′, or Carrington's for lat. 14° 46′.